6.3 Solar cell driving a load

a A Si solar cell of area 4 cm^2 is connected to drive a load R as in Figure 6.8 (a). It has the I-V characteristics in Figure 6.8 (b) under an illumination of 600 W m^{-2}. Suppose that the load is 20 Ω and it is used under a light intensity of 1 kW m^{-2}. What are the current and voltage in the circuit? What is the power delivered to the load? What is the efficiency of the solar cell in this circuit?

b What should the load be to obtain maximum power transfer from the solar cell to the load at 1 kW m^{-2} illumination. What is this load at 600 W m^{-2}?

c Consider using a number of such a solar cells to drive a calculator that needs a minimum of 3V and draws 3.0 mA at 3 - 4V. It is to be used indoors at a light intensity of about 400 W m^{-2}. How many solar cells would you need and how would you connect them? At what light intensity would the calculator stop working?

Solution

a The solar cell is used under an illumination of 1 kW m^{-2}. The short circuit current has to be scale up by 1000/600 = 1.67. Figure 6Q3-2 shows the solar cell characteristics scaled by a factor 1.67 along the current axis. The load line for R = 20 W and its intersection with the solar cell I-V characteristics at P which is the operating point P. Thus,

$$I¢ \approx 22.5 \text{ mA and } V¢ \approx 0.45 \text{ V}$$

The power delivered to the load is

$$P_{out} = I¢V¢ = (22.5\times10^{-3})(0.45\text{V}) = 0.0101\text{W, or } 10.1 \text{ mW.}$$

This is *not* the maximum power available from the solar cell. The input sun-light power is

$$P_{in} = \text{(Light Intensity)(Surface Area)}$$
$$= (1000 \text{ W m}^{-2})(4 \text{ cm}^2 ´ 10^{-4} \text{ m}^2/\text{cm}^2) = 0.4 \text{ W}$$

The efficiency is

$$\eta = 100\frac{P_{\text{out}}}{P_{\text{in}}} = 100\frac{0.010}{0.4} = 2.5\%$$

which is poor.

b Point M on Figure 6Q3-2 is probably close to the maximum efficiency point, $I¢ \approx 23.5$ mA and $V¢ \approx 0.44$ V. The load should be R = 18.7 W, close to the 20 W load. At 600 W m^{-2} illumination, the load has to be about 30 W as in Figure 6.8 (b). Thus, the maximum efficiency requires the load R to be decreased as the light intensity is increased. The fill factor is

$$\text{FF} = \frac{I_m V_m}{I_{sc}V_{oc}} = \frac{(23.5 \text{ mA})(0.44 \text{ V})}{(27 \text{ mA})(0.50 \text{ V})} \approx \mathbf{0.78}$$

c The solar cell is used under an illumination of 400 W m^{-2}. The short circuit current has to be scale up by 400/600 = 0.67. Figure 6Q3-2 shows the solar cell characteristics scaled by a factor 0.67 along the current axis. Suppose we have N identical cells in series, and the voltage across the calculator is $V_{\text{calculator}}$. The current taken by the calculator is 3 mA in the voltage range 3 to 4 V and the calculator stops working when $V_{\text{calculator}} < 3$ V. The cells are in series so each has the same current and equal to 3 mA, marked as $I¢$ in Figure 6Q-2. The voltage across one cell will be $V¢ = V_{\text{calculator}}/N$. which is marked in Figure 6Q3-2. V¢ = 0.46 V. Minimum number of solar cells in series = N = 3 / 0.46 = 6.5 or 7 cells , since you must choose the nearest higher integer.

If we want the calculator continue to work under low intensity levels, then we can connect more cells in series until we reach about 4 V; N = 4 V / 0.46 V = 8.7 or 9 cells in series.

The easiest estimate for the minimum required light intensity is the following: The calculator will stop working when the light intensity cannot provide energy for the solar cell to deliver the 3 mA calculator current. The short circuit current at 400 W m^{-2} is 11 mA in Figure 6Q3-2. Thus

$$\text{Minimum light intensity} = \frac{3\text{ mA}}{11\text{ mA}}400\ \ \text{W m}^{-2}$$ **=109 W m^{-2}**

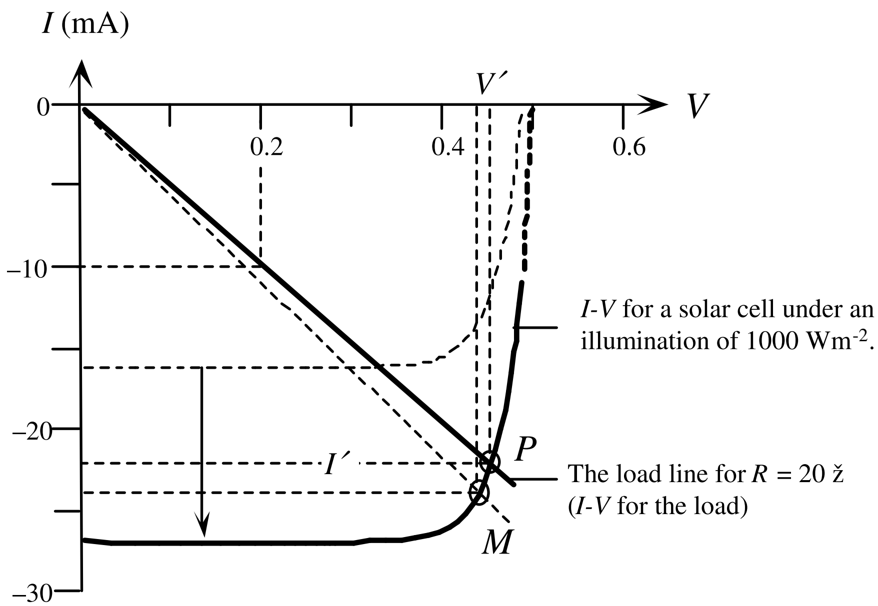


Figure 6Q3-1

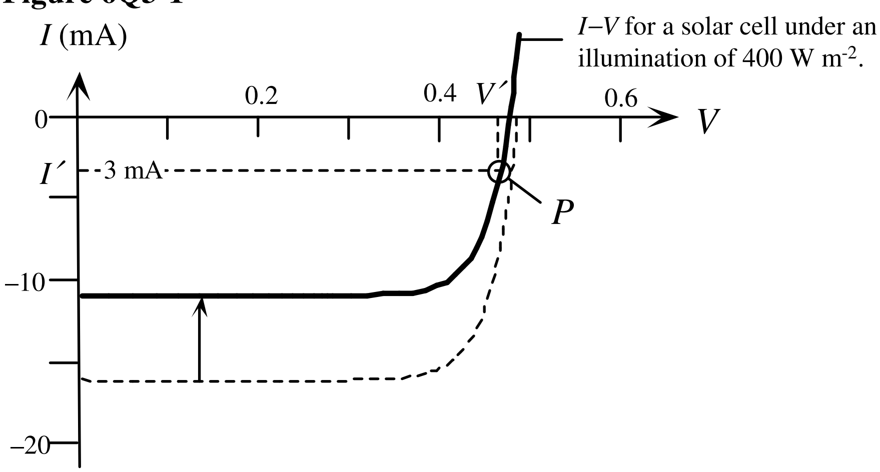


Figure 6Q3-2

6.4 Open circuit voltage A solar cell under an illumination of 100 W m^{-2} has a short circuit current I_{sc} of 50 mA and an open circuit output voltage V_{oc}, of 0.55V. What are the short circuit current and open circuit voltages when the light intensity is halved?

Solution

The short circuit current is the photocurrent so that at

$$I_{sc2} = I_{sc1}\left(\frac{I_2}{I_1}\right) = (50\ \ \text{mA})\left(\frac{50\ \ \text{W m}^{-2}}{100\ \ \text{W m}^{-2}}\right)$$ = 25 mA

Assuming $n = 1$, the new open circuit voltage is

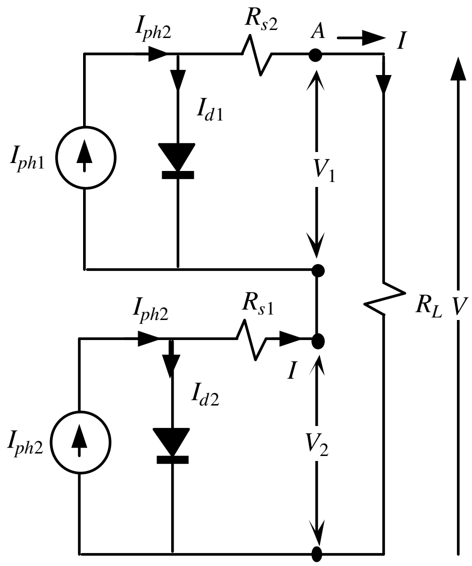
$$V_{oc2} = V_{oc1} + \frac{nk_BT}{e}\ln\left(\frac{I_2}{I_1}\right) = 0.55 + 1(0.0259)\ln(0.5)$$ **= 0.508 V**

Assuming $n = 2$, the new open circuit voltage is

$$V_{oc2} = V_{oc1} + \frac{nk_BT}{e}\ln\left(\frac{I_2}{I_1}\right) = 0.55 + 2(0.0259)\ln(0.5)$$
$$= \mathbf{0.467\ V}$$

6.7 Series connected solar cells Consider two odd solar cells. Cell 1 has I_{o1} = 25´10^{-6} mA, n_1 = 1.5, R_{s1} = 10 W and cell 2 has I_{o2} = 1´10^{-7} mA, n_2 = 1, R_{s2} = 50 W. The illumination is such that I_{ph1} = 10 mA and I_{ph2} = 15 ma. Plot the individual *I-V* characteristics and the *I-V* characteristics of the two cells in series. Find the maximum power that can be delivered by each cell and two cells in series. Find the corresponding voltages and currents at the maximum power point. What is your conclusions?

Solution



Two different solar cells in series

Figure 6Q7-1

The equivalent circuit is shown in Figure 6Q7-1. The current through both the devices has to be the same. Thus, for cell 1

$$I = -I_{ph1} + I_{o1}\left[\exp\left(\frac{V_{d1}}{n_1V_T}\right) - 1\right] = -I_{ph1} + I_{o1}\left[\exp\left(\frac{V_1 - IR_{s1}}{n_1V_T}\right) - 1\right]$$

∴
$$\frac{V_1 - IR_{s1}}{n_1V_T} = \ln\left(\frac{I + I_{ph1}}{I_{o1}} - 1\right)$$

∴
$$V_1 = n_1V_T\ln\left(\frac{I + I_{ph1}}{I_{o1}} - 1\right) + IR_{s1}$$

For cell 2
$$I = -I_{ph1} + I_{o2}\left[\exp\left(\frac{V_{d2}}{n_2V_T}\right) - 1\right] = -I_{ph2} + I_{o2}\left[\exp\left(\frac{V_2 - IR_{s2}}{n_2V_T}\right) - 1\right]$$

∴
$$V_2 = n_2V_T\ln\left(\frac{I + I_{ph2}}{I_{o2}} + 1\right) + IR_{s2}$$

But $V = V_1 + V_2$

∴
$$V = n_1V_T\ln\left(\frac{I + I_{ph1}}{I_{o1}} + 1\right) + IR_{s1} + n_2V_T\ln\left(\frac{I + I_{ph2}}{I_{o2}} + 1\right) + IR_{s2}$$

We can now substitute I_{o1} = 25´10^{-6} mA, n_1 = 1.5, R_{s1} = 10 W, I_{o2} = 1´10^{-7} mA, n_2 = 1, R_{s2} = 50 W and then plot V vs. I (rather I vs. V since we can calculate V from I) for each cell and the two cells in series as in Figure 6Q7-2. Notice that the short circuit is determined by the smallest I_{sc} cell. The total open circuit voltage is the sum of the two.

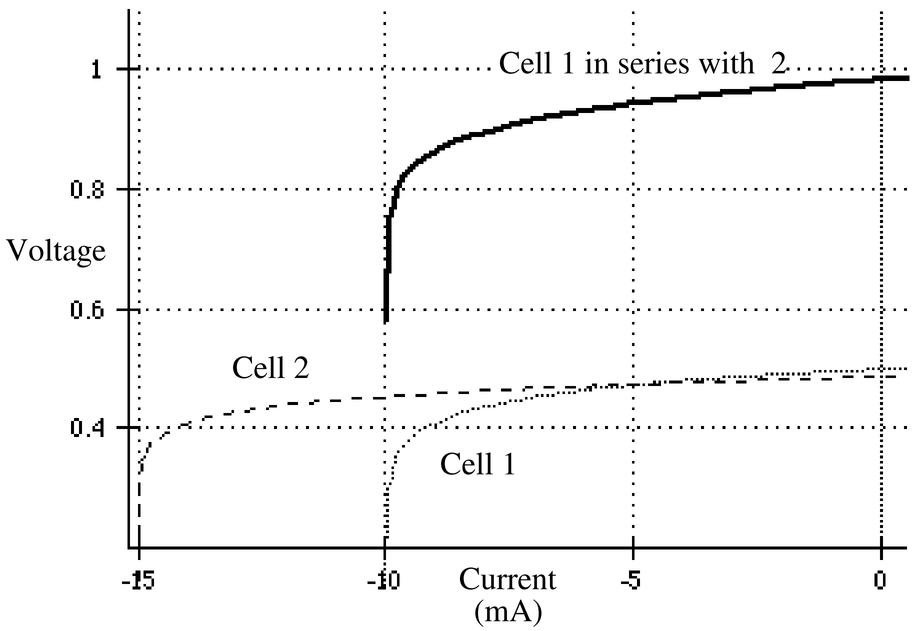


Figure 6Q7-2

6.9 Solar cell efficiency The fill factor FF of a solar cell is given by the empirical expression

$$\mathrm{FF} \approx \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 2}$$

where $v_{oc} = V_{oc}/(nk_BT/e)$ is the normalized open circuit voltage (normalized with respect to the thermal temperature k_BT/e). The maximum power output from a solar cell is

$$P = \mathrm{FF} I_{sc} V_{oc}$$

Taking V_{oc} = 0.58 V and $I_{sc} = I_{ph}$ = 35 mA cm^{-2}, calculate the power available per unit area of solar cell at room temperature 20 °C, at -40 °C and at 40 °C

Solution

The open circuit voltage depends on the temperature whereas I_{sc} has very little temperature dependence. Use

$$V'_{oc} = V_{oc}\left(\frac{T'}{T}\right) + \frac{E_g}{e}\left(1 - \frac{T'}{T}\right) \quad (1)$$

to calculate the V_{oc} at different temperature given V_{oc} at one temperature. Then calculate v_{oc} using

$$v_{oc} = V_{oc}/(nk_BT/e) \quad (2)$$

then FF using

$$\mathrm{FF} \approx \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 2} \quad (3)$$

and then P using

$$P = \mathrm{FF} I_{sc} V_{oc} \quad (4)$$

as summarized in Table 6Q9-1.

Table 6Q9-1

$n = 1$

	I_{sc} mA/cm^2	V_{oc}; Eq. (1)	v_{oc}; Eq. (2)	FF; Eq. (3)	P mW cm^{-2}; Eq. (4)
20 °C	35	0.580 V	22.97	0.793	16.03
-40 °C	35	0.686 V	34.19	0.847	20.34
40 °C	35	0.545	20.19	0.773	14.73

$n = 2$

	I_{sc} mA/cm^2	V_{oc}; Eq. (1)	v_{oc}; Eq. (2)	FF; Eq. (3)	P mW cm^{-2}; Eq. (4)
20 °C	35	0.580 V	11.49	0.666	13.52
-40 °C	35	0.686 V	17.10	0.744	17.90
40 °C	35	0.545	10.09	0.638	12.15

Conclusions: $n = 2$ case has a lower FF and also lower power delivery.

NOTE: The temperature dependence of the open circuit voltage V_{oc} was derived in the text as

$$V'_{oc} = V_{oc}\left(\frac{T'}{T}\right) + \frac{E_g}{e}\left(1 - \frac{T'}{T}\right)$$

This expression is valid whether n is 1 or 2. Recall that $n = 1$ represents diffusion in the neutral regions and $n = 2$ is recombination in the space charge layer. In the $n = 1$ case I_o µ n_i^2 and in the $n = 2$ case I_o µ n_i, thus in general

$$I_o \text{ µ } n_i^{2/n}$$

Consider the open circuit voltage,

$$V_{oc} = \frac{nk_BT}{e}\ln\left(\frac{KI}{I_o}\right) \quad \text{or} \quad \frac{eV_{oc}}{nk_BT} = \ln\left(\frac{KI}{I_o}\right)$$

At two different temperatures T_1 and T_2 but at the same illumination level, by subtraction,

$$\frac{eV_{oc2}}{nk_BT_2} - \frac{eV_{oc1}}{nk_BT_1} = \ln\left(\frac{I_{o1}}{I_{o2}}\right) \approx \ln\left(\frac{n_{i1}^{2/n}}{n_{i2}^{2/n}}\right)$$

where the subscripts 1 and 2 refer to the temperatures T_1 or T_2 respectively.

We can substitute $n_i^{2/n} = (N_cN_v)^{1/2}\exp(-E_g/nk_BT)$ and neglect the temperature dependences of N_c and N_v compared with the exponential part to obtain,

$$\frac{eV_{oc2}}{nk_BT_2} - \frac{eV_{oc1}}{nk_BT_1} = \frac{E_g}{nk_B}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Rearranging for V_{oc2} in terms of other parameters we find,

$$V_{oc2} = V_{oc1}\left(\frac{T_2}{T_1}\right) + \frac{E_g}{e}\left(1 - \frac{T_2}{T_1}\right)$$